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A Heart Chamber Model Using a Capacitance Function Combined With the Navier–Stokes Equations

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ABSTRACT

We present a simplified mathematical and computational model of blood flow through a chamber of a healthy human heart, with a particular description of the left ventricle. We use fixed representations of the domain and mesh, assume blood to be an incompressible Newtonian fluid, and simulate the chamber volume variation by introducing a capacitance function, which is space and time-dependent. The inclusion of this capacitance function gives rise to a model whose differential form resembles the compressible Navier–Stokes equations. The numerical methodology is based on stable Galerkin mixed finite element formulations posed on velocity and pressure. Convergence studies for the two-dimensional steady-state Stokes equation with spatially-varying capacitance indicate optimal convergence orders when combining continuous biquadratic velocities with continuous bilinear pressures (Taylor–Hood elements) or discontinuous unmapped linear pressures. On the other hand, we observed loss of optimality with discontinuous linear mapped approximations for this field, as expected for the classical Stokes problem. Additional convergence studies indicate that the optimal convergence properties are preserved in the resolution of the nonlinear model, for the case of smooth solutions. Finally, we present simulations of cardiac cycles in an idealized human left ventricle in two and three dimensions. The results are meaningful given the model simplicity and open the possibility for future extensions of the proposed methodology.